Further Comments on "Modes of Elliptical Waveguides: A Correction"

J. C. Wiltse and T. H. Gfroerer

In the above paper[1], the authors discuss the TM_{01} mode in elliptical waveguides and describe an error in the field configurations plotted in the early article by Chu [1] and repeated in the book by Marcuvitz [2]. The authors comment "that the error had apparently gone undetected for some five decades." This is not correct, and in fact, this and other errors in the Chu article have been discussed by numerous authors over a period of many years.

The specific error dealing with the TM_{01} mode was first pointed out by W. Krank in 1962 [3]. In 1964 Piefke [4] published a detailed analysis of the modes and gave plots of the correct field configurations (see Fig. 3(g), p. 261, for the TM_{01} mode). Kretzschmar also published several articles (two in these transactions) on the subject in 1970 [5], 1971 [6], and 1972 [7]. In particular, in the 1971 article he specifically pointed out the Chu error for the TM_{01} mode fields and showed plots of Chu's configuration and the correct version. The arguments given by Goldberg *et al.* for the corrected field configuration are similar to Kretzschmar's discussion [6].

In the paper by Goldberg *et al.* they state (p. 1605, Section III): . . . "the exact solutions (method 1) clearly lie off Chu's curves for the higher eccentricity. This would suggest that the accuracy of either the tables or the truncated expansions used by Chu decrease in the limit of large q and small ξ." Lewin and Al-Hariri published a paper in 1974 which already demonstrated that the expansion used by Chu is not valid unless ξ is large. In fact, Chu himself concedes the assumption of large ξ (p. 588, top right column). Also included in Lewin and Al-Hariri's paper is a correction of the error.

Over the years, several other authors have discussed various errors in the original Chu paper. These include limitations in his results because of his choice of particular asymptotic formulas for the radial Mathieu functions, and errors in his solutions for attenuation and surface impedance [8]–[11]. Reference [11] contains a compilation of previous conclusions. In summary, the results given by Goldberg *et al.*, have already been described in the literature, and in addition, the earlier papers contain more information about mode configurations and propagation characteristics.

This work was supported in part by the U.S. Army Research Office. Opinions, interpretations, conclusions, and recommendations are those of the authors and are not necessarily endorsed by the U.S. Army.

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Manuscript received May 9, 1991.
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IEEE Log Number 9103914.

[1]D. A. Goldberg, L. J. Laslett, and R. A. Rimmer, *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1603–1608, Nov. 1990.

Authors' Reply[2]

David A. Goldberg, L. Jackson Laslett, and Robert A. Rimmer

This letter is in reply to the comments of Dr. J. B. Davies and of Drs. J. C. Wiltse and T. H. Gfroerer on our paper. Despite our modest intentions, our paper seems to have created something of a tempest in an (elliptical) teapot. We deeply regret this.

Addressing the comments of Dr. Davies first, we would like to thank him for calling attention to several articles dealing with the modes of elliptical waveguides which we had omitted from our bibliography. Another work which we have become aware of since the publication of our article (which is also included in the Wiltse and Gfroerer bibliography) is

> B. Rembold, "Elliptische hohlleiter, tafeln für die Grenzwellenlängen und Dämpfungskonstanten," *Archiv für Electronik and Übertragungstechnik*, vol. 29, pp. 449–453, Nov. 1975.

Our failure to be fully conversant with the relevant literature is at least partly due to the fact, evident from our biographies, that none of the authors is a regular practitioner in the microwave field.

Having said that, we feel obliged to point out that the main point that we wished to make in our paper, was *not* any disagreement

[2]Manuscript received June 5, 1991.
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with Chu's eigenvalues (which as we noted proved remarkably accurate for the case of $e = 0.75$), but rather that the correct field pattern for the TM_{01} mode was *qualitatively* radically different from Chu's; neither Rembold's paper, nor any of those referred to by Dr. Davies, addresses the question of the field shape of the TM_{01} mode. We should also note that, having found Chu's figure reproduced in the 1986 reprinting of Marcuvitz's book (which, we are informed by colleagues, is something of a "bible" in the field), with no mention of any associated erratum, we restricted our literature search to papers published after 1985, and, in any case, to papers showing the actual field shapes (we found none). Although it was not the purpose of our note to present a comprehensive study of the elliptical waveguide, we nonetheless welcome Dr. Davies' suggestions for remedying the noted deficiencies in our bibliography.

Finally a note on the "demystifying" of Mathieu functions. It has been our experience that not all of our colleagues are as conversant with these functions as Dr. Davies obviously is, and our remark was intended as a somewhat light-handed way of acknowledging this fact. We regret any offense that may have been given; none was intended.

Many of the above remarks are equally applicable to the comments of Wiltse and Gfroerer. In particular, [11] in their article, which is said to summarize the various corrections to the Chu paper, refers exclusively to wave-impedance calculations and makes no reference whatsoever to field shapes.

The work described in [6] in their article (the 1971 Kretzschmar paper) is another matter. As we stated in replying to Dr. Davies's comments, we relied on what a number of electrical engineering colleagues advised us was the standard reference work (Marcuvitz's *Waveguide Handbook*,) and only searched the *subsequent* literature, so we were indeed ignorant of Kretszchmar's work on the error in the field shapes. While it does not fully exonerate us, we find our ignorance of this subsequent work places us in rather learned company: In addition to the three referees of our paper, we would add Dr. Julius Stratton (Chu's thesis advisor), Dr. N. Marcuvitz, and, apparently, Kretzschmar's co-author on a 1972 paper on elliptical waveguides, one J. B. Davies. (Whether we should be similarly faulted for our ignorance of an unpublished 1962 thesis from the Aachen Technische Hochschule, we leave to the judgment of your readers.)

Part of the difficulty seems to be the lack of "standard references" which are up to date; despite the apparent "textbook" nature of the elliptical waveguide problem, none of the sources for the corrections referred to by either Davies or Wiltse and Gfroerer is such a source. In fact one of our main motivations in writing the paper was to point out a qualitative error that had persisted in the latest edition of one of the most heavily relied on standard sources. Indeed, our decision to publish in IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, rather than the *Journal of Applied Physics* (in which Chu's paper originally appeared), was to bring the correction to the attention of the widest possible audience. We feel fairly certain that the combination of our article and the lively correspondence it has generated will achieve that goal, if not precisely in the way originally intended.

In closing, we would like to thank the authors of the two letters for their interest and comments, and, since we have not yet explicitly done so, to extend our apologies to Dr. Kretzschmar for inadvertently taking the credit which is rightfully his.

Comments on "Full-Wave Analysis of Discontinuities in Planar Waveguides by the Method of Lines Using a Source Approach"

Ling Chen

I think there are some mistakes in the above paper[1]. The method presented in that article is unavailable. Because (2) in the article should be

$$\psi_0^{e+} = -\psi_0^{e-} = \frac{\psi_0^e}{1-r} \quad \text{and} \quad \psi_0^{h+} = \psi_0^{h-} = \frac{\psi_0^h}{1+r}. \tag{2}$$

According to this, (3) should be

$$\left.\frac{\partial \psi^h}{\partial z}\right|_{z=0} = -j\beta \frac{1-r}{1+r} \psi_0^h. \tag{3}$$

Equation (15) should be

$$\left[h^2 \frac{\partial^2 \psi^{h2}}{\partial z^2}\right] r_{zh}^{-1} = \left[h \frac{\partial \psi^h}{\partial z}\right] r_{ze}^{-1} D_z^t - \left.\left[h \frac{\partial \psi^h}{\partial z}\right]\right|_{sz=0} r_{zh1}$$
$$= -\psi^h r_{zh}^{-1} D_z D_z^t + j\beta \frac{1-r}{1+r} h \psi_0^h r_{zh1}. \tag{15}$$

Equation (16) should be

$$r_{xh}^{-1}\left[h^2 \frac{\partial^2 \psi^h}{\partial z^2}\right] r_{xh}^{-1} \rightarrow -\phi^h D_z D_z^t + j\beta \frac{1-r}{1+r} h \phi_0^h$$
$$= \phi^h D_{zz}^{ht} + j\beta \frac{1-r}{1+r} h \phi_0^h. \tag{16}$$

Equation (32) should be

$$\frac{d^2 \phi^h}{dy^2} + \frac{D_{xx}^h \phi^h}{h^2} + \frac{\phi^h D_{zz}^{ht}}{h^2} + \epsilon_r k_0^2 \phi^h = \frac{-j\beta h \phi_0^h}{h^2} \frac{1-r}{1+r}. \tag{32}$$

Equation (33) should be

$$[V_p]_{ik} = \frac{-j\beta h [V_0]_{ik}}{([k_0]_i^2 - k_{ik}^2) h^2} \frac{1-r}{1+r}. \tag{33}$$

Equation (34) should be

$$h \left.\frac{d[V]_{ik}}{dy}\right|_{y=0} = -[\gamma^h]_{ik} [V]_{ik} + j\beta \frac{1-r}{1+r} h [\gamma_q^h]_{ik} [V_0]_{ik}. \tag{34}$$

Therefore, $\tilde{Z}_q$ in (37) and $Z_{q,\text{red}}$ in (38) are related to the unsolved parameter r. So, the current distribution cannot be obtained from (38), the reflection coefficient and the normalized input impedance cannot be obtained.

Manuscript received May 16, 1991.

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IEEE Log Number 9103911.

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